Transformations

8

At the Amusement Park

translation

translation image

translation arrow

reflection

reflection image

line of reflection

rotation

rotation image

quarter turn

half turn

three-quarter turn

clockwise

counterclockwise

point of rotation

transformations

Learning Goals

- translate, reflect, and rotate a shape
- draw and describe images after transformations
- identify a transformation

Look at the map of the amusement park.

- What rides do you see?
- How do people move on each ride?
- What is your favourite ride at an amusement park? How do you move on that ride?

294

295

1 Translations

A firefighter slides down a pole.

A flag slides up a pole.

A child slides down a playground slide.

Which other ways do people or objects slide?

Explore 👫

You will need Pattern Blocks, dot paper, and a ruler.

➤ Choose a Pattern Block.
   Place it on the dot paper.
   Trace the block.
   Slide the block in a straight line, in any direction.
   Do not turn the block.
   Use a ruler if it helps.
   Trace the block in its new position.
   How do the two positions of the block compare?

➤ Take turns to move a block and describe how its two positions compare.

Show and Share

Compare drawings with another pair of classmates.
How do the original shape and the shape in its new position compare?

Connect

When a shape moves along a straight line, without turning, it is *translated* from one position to another.
The movement is a **translation** or a *slide*.

When we draw the shape in its new position, we draw a **translation image** of the shape.

The translation is described by the numbers of squares moved right or left and up or down.
The translation below is:
5 squares right and 4 squares down.
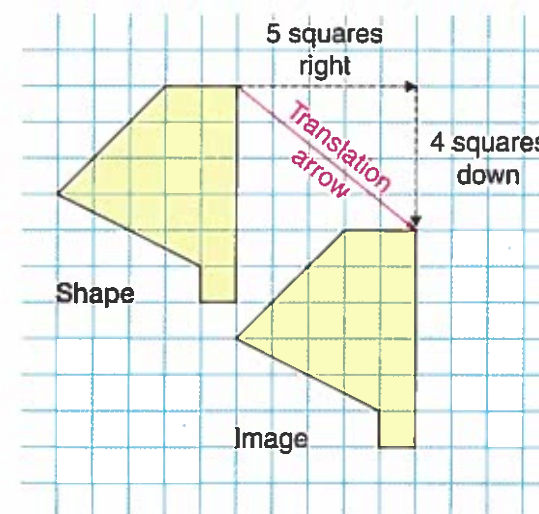
We say how many squares right or left before we say how many up or down.

5 squares right

Translation arrow

4 squares down

Shape

Image

If we cannot translate the shape, we trace the shape, then translate the tracing.

The **translation arrow** shows how the shape moved.
The arrow joins matching points on the shape and its image.

A shape and its translation image have the same orientation; that is, they face the same way.

1. Copy each shape on grid paper. Use tracing paper.
   Translate the shape using the given translation.
   Draw the image and a translation arrow.
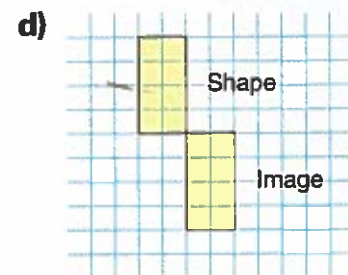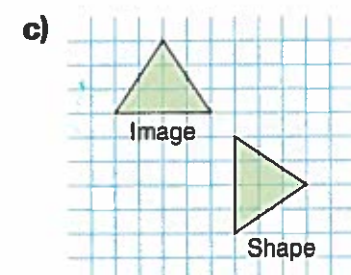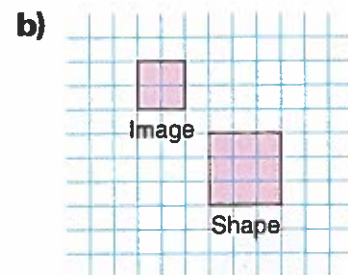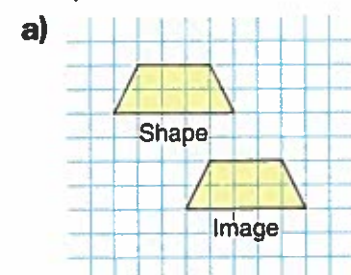   Describe the position and orientation of the image.
   a) 7 squares left and   b) 5 squares right   c) 3 squares left and
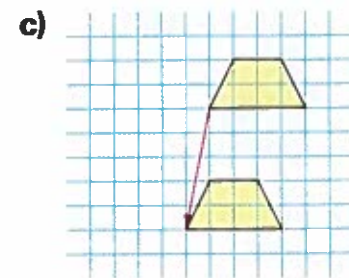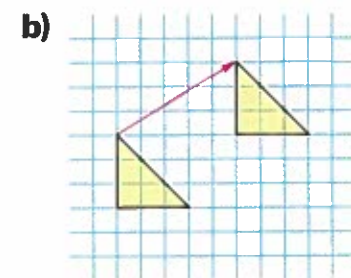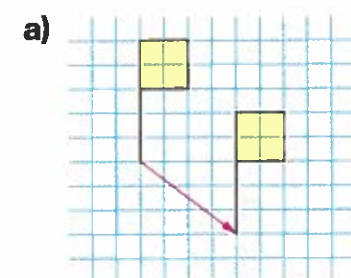   3 squares up   and 4 squares down   6 squares down
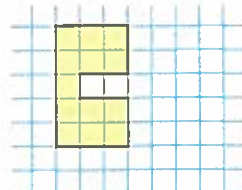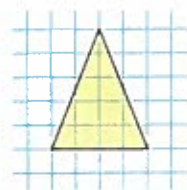
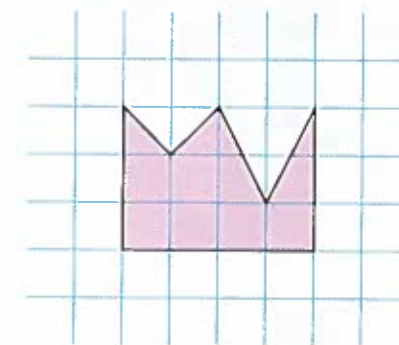2. Does each picture show a translation?
   How do you know?
   If a picture does show a translation, describe it.
   a)

   Shape

   Image

   b)

   Image

   Shape

   c)

   Image

   Shape

   d)

   Shape

   Image

3. Write the translation that moved each shape to its image.
   a)   b)   c)

4. a) Draw this shape on grid paper.
      Predict where the image will be after
      this translation:
      3 squares left and 5 squares up
      Draw the image to check your prediction.
   b) Draw the shape again.
      Predict where the image will be after
      this translation:
      5 squares left and 3 squares up
   Draw the image to check your prediction.

5. Draw a shape on dot paper.
   Translate the shape in any direction.
   Draw its image.
   Record the translation.
   Describe the position and orientation of the image.

6. Draw two identical shapes in two different places
   on grid paper.
   Make sure the shapes have the same orientation.
   Label one shape "Image" and the other "Shape".
   Which translation will move the shape to its image?

7. Copy these shapes on grid paper.

   A

   B
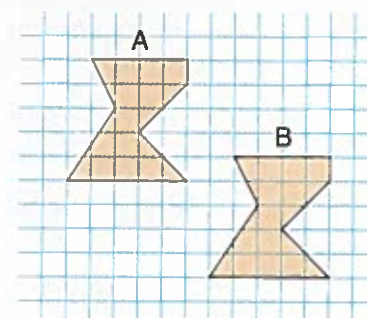
   a) Describe which translation moves Shape A to Shape B.
   b) Describe which translation moves Shape B to Shape A.

Reflect

Use grid paper.
Draw a shape and its translation image.
Explain how you know your picture shows a translation.

Strategies Toolkit

the tortoise had a race.
times around the running track.
times around in 1 h,
for a rest.
not stop.
go once around the track.
up after 4 h.
race?

Share

solved the problem.

Strategies
- **Make a table.**
- **Use a model.**
- **Draw a picture.**
- **Solve a simpler problem.**
- **Work backward.**
- **Guess and test.**
- **Make an organized list.**
- **Use a pattern.**

bottom of a well.
ery day, but it slides
ght.
ee p.
does it take the snail
well?

What do you know?
  Each day, the snail climbs
  2 m up the well.
  Each night, the snail slides back 1 m.
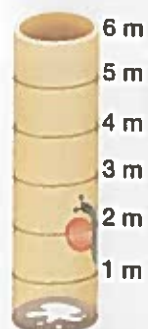 The snail has to climb 6 m to get out.

Think of a strategy to help you solve the problem.
You can **draw a picture.**
Show where the snail is each day.

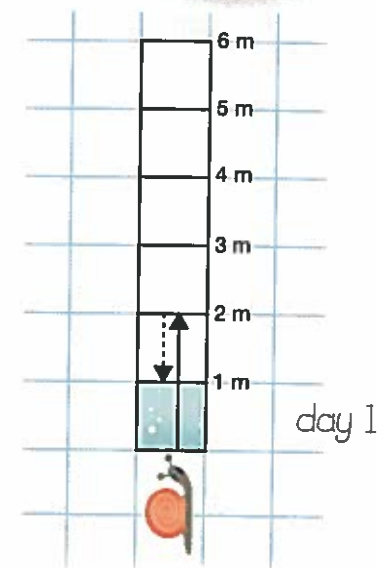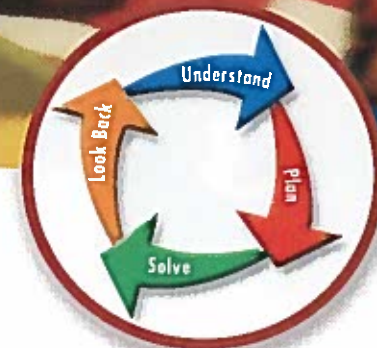Interpret a problem and select an appropriate strategy.

Use grid paper to record how far the snail moves.
Use a different colour for each day.
Count the days when the snail reaches the top of the well.
When does the snail get out of the well?

Write a similar problem.
Have a classmate solve your problem.

day 1

Practice

Choose one of the **Strategies**

1. Shannon is shorter than Bruce.
   Olivia is shorter than Alex but taller than Bruce.
   Who is the tallest? Shortest?

2. Hannah and Liam are using a compass.
   They move 30 m north, then 30 m west, and then 30 m south.
   Which direction do Hannah and Liam go to get back to where they started?
   How far must they go?

Reflect

How does drawing a picture help you to solve a problem?

3 Reflections

A **reflection** can be used to make an interesting picture.
Is this person floating above the ground?
Where else do you see reflections?

Explore

You will need Pattern Blocks, dot paper, a ruler, and a Mira.

➤ Draw a line through the centre of the dot paper.
Place a Mira on this line.

➤ Place a block on one side of the line.
Your partner places her block on the image she sees in the Mira.

➤ Take turns to place one block, then another block on its image.
Each time, describe the position and orientation of the image.

➤ Take turns to draw around a block and its image.
Draw around blocks that touch the Mira line.
Draw around blocks that cross the Mira line.
In each case, how does the shape compare with its image?

Show and Share

Compare your pictures with those of another pair of classmates.
How is each shape and its image placed with respect to the Mira line?

Connect

When a shape is *reflected* in a mirror, we see a **reflection image**.

The line segment that joins a point to its image is perpendicular to the **line of reflection**.

A point and its image are the same distance from the line of reflection.

A shape and its reflection image have opposite orientations; that is, they face opposite ways.
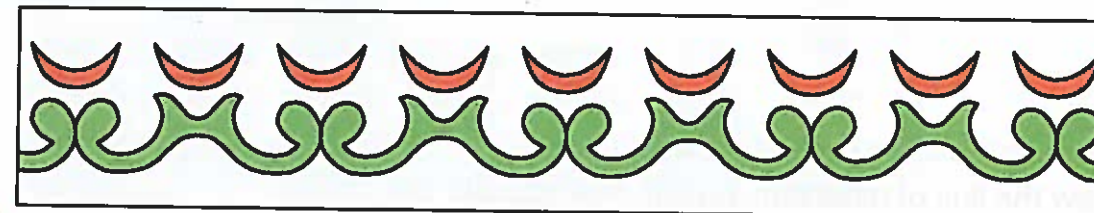
A reflection is sometimes called a *flip*.
When a shape is reflected, it is flipped over.
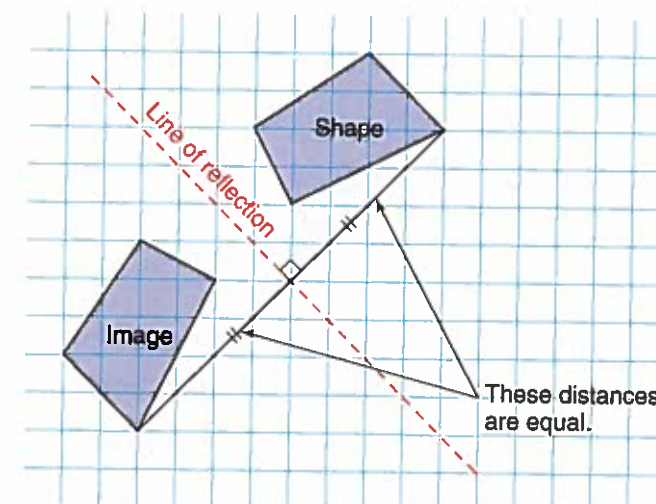

Math Link

Your World

Many patterns and designs show a shape and its reflection images.
Identify a shape and its reflection images in this design.
Where are the lines of reflection?

Use a Mira when it helps.

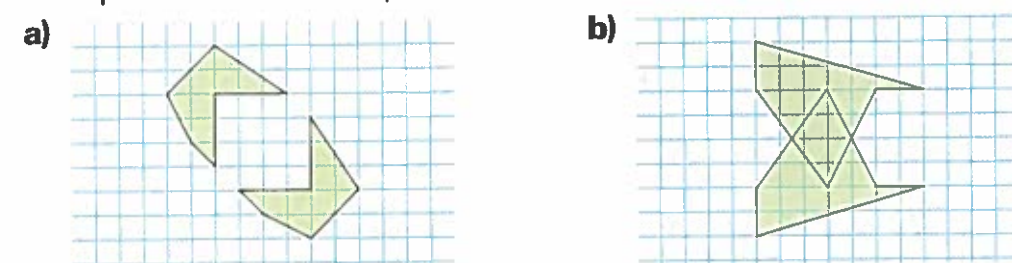1. Copy each shape and line of reflection on grid paper.
   Draw each reflection image.
   Describe the position and orientation of the image.

   a)
   Line of reflection

   b)
   Line of reflection

   c)
   Line of reflection

   d)
   Line of reflection
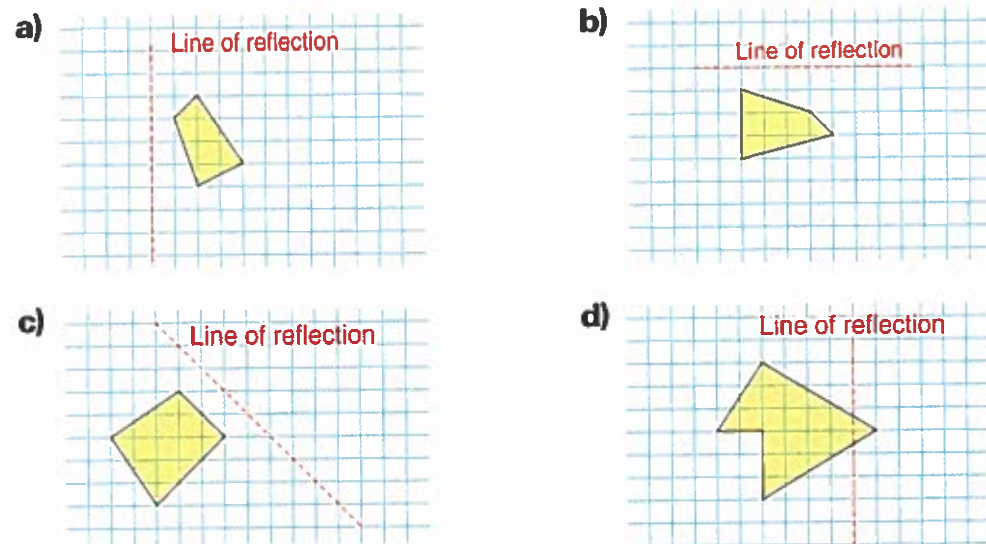
2. Which pictures show a reflection? How do you know?
   Describe where the line of reflection is.

   a)   b)   c)
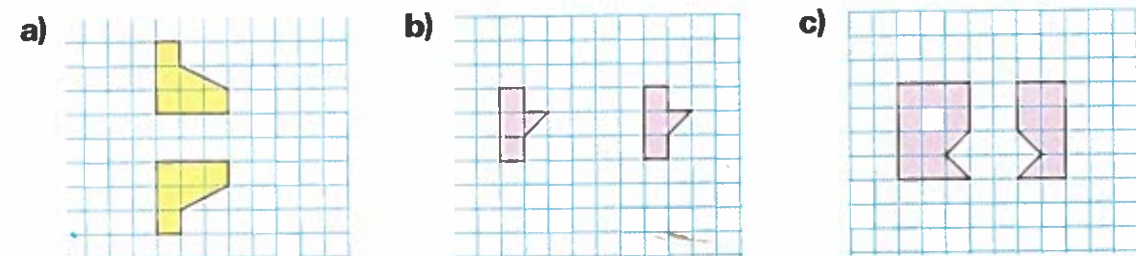
3. In question 2, do any pictures show a translation?
   If so, describe the translation.

4. Each picture shows a shape and its reflection image.

   a)   b)

   Copy each picture on grid paper.
   Draw the line of reflection. Explain how you did this.
   How do you know the line of reflection is drawn correctly?

5. Copy each shape and line of reflection on dot paper.
   Predict where each reflection image will be.
   Draw each image to check your prediction.

   a)   b)
   Line of reflection   Line of reflection

6. Print the letters of the alphabet as capital letters.
   a) Draw a horizontal line above each letter.
      Place a Mira on the line.
      Which letters look the same in the Mira?
   b) Draw a vertical line beside each letter.
      Place a Mira on the line.
      Which letters look the same in the Mira?
   c) Create three words whose images
      read the same as the words when
      a Mira is placed above the letters.

7. Draw a shape on dot paper.
   Draw and label a line of reflection.
   Draw the image of the shape in the line of reflection.
   a) Use a ruler.
      Join two matching points on the shape and its image.
   b) Use a ruler.
      Measure the distance between each point and the line of reflection.
      What do you notice?
   c) What do you notice about the angle between the line you drew
      in part a and the line of reflection?
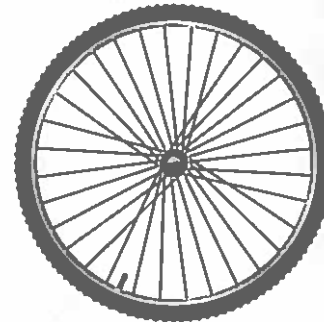   Show your work. Explain your thinking.

Reflect

How are a translation and a reflection alike?
Draw a shape and its image that could show both
a reflection and a translation.

4 Rotations

A bicycle wheel turns about the centre of the wheel.

What other examples are there of things that turn? Explain how they turn.

Explore

You will need several pieces of paper, tracing paper, a ruler, a compass, and scissors.

➤ Use a ruler.
Draw a shape with straight sides in the centre of a piece of paper.
➤ Use tracing paper.
Draw an identical shape on another piece of paper.
Cut out this shape.
Place it on top of the first shape you drew.
➤ Put your compass point at one vertex.
Turn the shape to a new position.
Draw the shape in its new position.
Label this shape Image A.
➤ Return your shape to its original position.
Turn the shape in the opposite direction.
Draw the shape in its new position.
Label this shape Image B.
➤ In each case, how do the positions of the original shape and its image compare?

Show and Share

Compare your picture and ideas with another pair of classmates. Did you have the same ideas about how a shape compares with its image after a rotation? Explain.

Connect

When a shape turns about a point, it is *rotated* from one position to another.

The movement is a **rotation**, or *turn*. When we draw the shape in its new position, we draw a **rotation image** of the shape.

After 1 complete turn, a shape is back to where it started.

When the minute hand on a clock moves from 12 to 3, it moves a **quarter turn**.
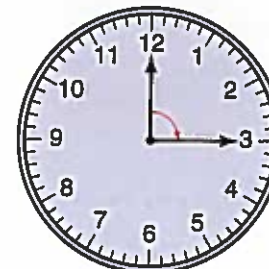
When the minute hand moves from 12 to 6, it moves a **half turn**.

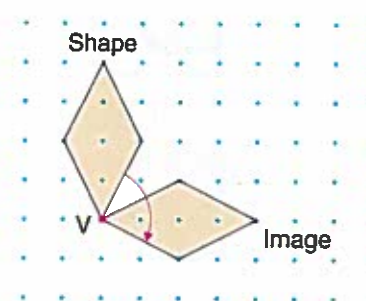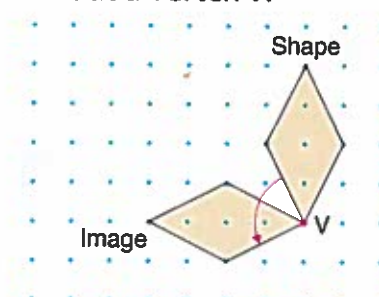When the minute hand moves from 12 to 9, it moves a **three-quarter turn**.

$\frac{1}{4}$ turn

$\frac{1}{2}$ turn
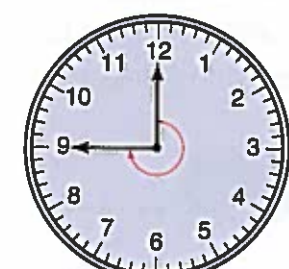
$\frac{3}{4}$ turn

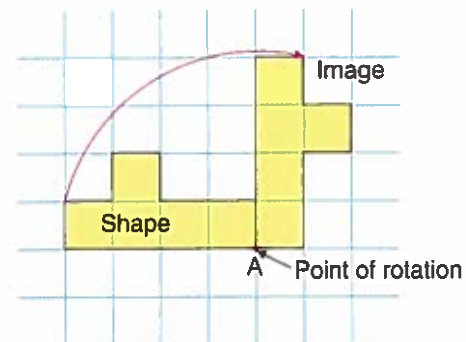A shape can rotate **clockwise** about a vertex V:

Shape

V

Image

A shape can rotate **counterclockwise** about a vertex V:
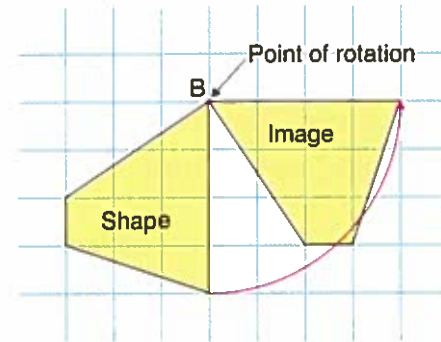
Shape

Image

V

Any turn less than 1 complete turn is a fraction of a turn clockwise or counterclockwise.

This shape has rotated a $\frac{1}{4}$ turn clockwise, about vertex A.
This point is called the **point of rotation**.

This shape has rotated a $\frac{1}{4}$ turn counterclockwise, about vertex B.

A rotation is described by:

- the direction of the turn (clockwise or counterclockwise),

- the fraction of the turn, and

- the point of rotation

A shape and its rotation image have different orientations. The shape and its image face different ways for any rotation that is less than 1 complete turn.

> A reflection, a rotation, and a translation are **transformations**.

Practice

1. Copy each shape below on grid paper. For each shape:
   - Rotate the shape about vertex V, using the rotation given.
   - Draw the rotation image.
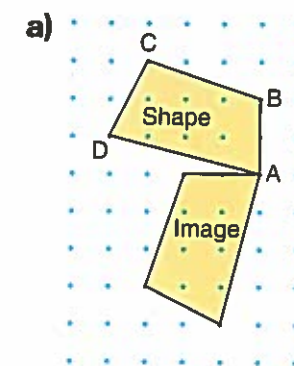   - Describe the position and orientation of the image.

   a) a $\frac{1}{4}$ turn counterclockwise
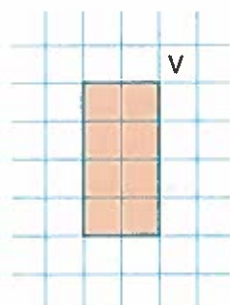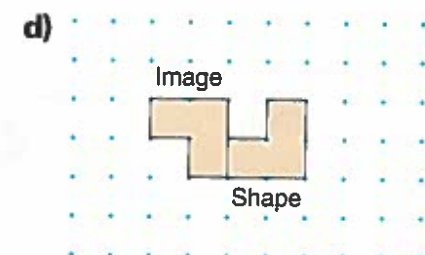

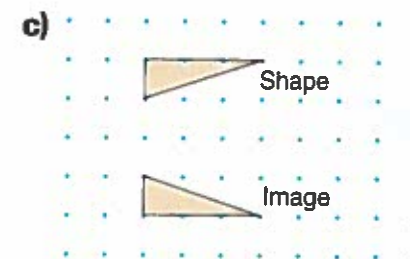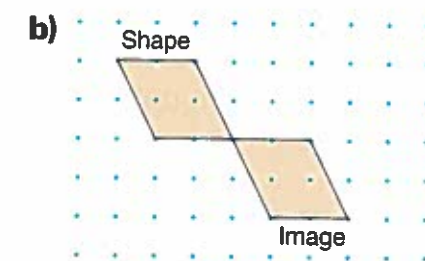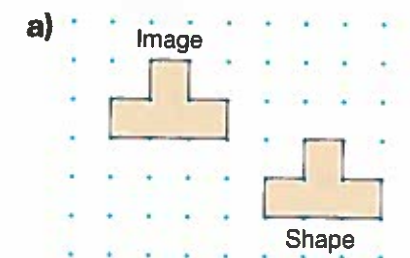   b) a $\frac{1}{2}$ turn clockwise
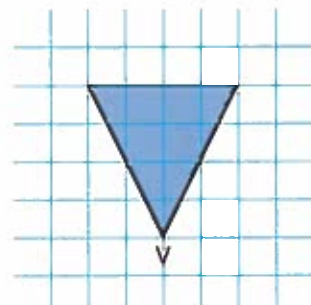
   c) a $\frac{3}{4}$ turn clockwise
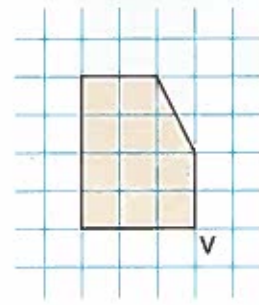
2. Each picture below shows a shape and its rotation image. Describe the rotation. Include the direction of the turn.

   a)

   b)
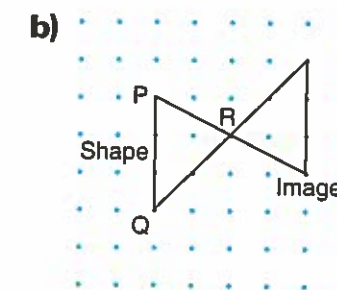
3. Which pictures show a rotation? How do you know? Describe the rotation.

   a)


   b)

   c)

   d)

4. Did any of the pictures in question 3 show a translation? A reflection? If so, identify the picture and describe the transformation.

5. Copy this shape.
   Trace the shape on tracing paper.
   Use the tracing to rotate the shape.
   Predict the position of the image after each rotation below.
   Draw each image to check your prediction.

   a) a $\frac{1}{4}$ turn clockwise about vertex A

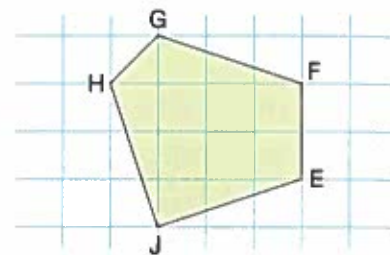   b) a $\frac{3}{4}$ turn counterclockwise about vertex A
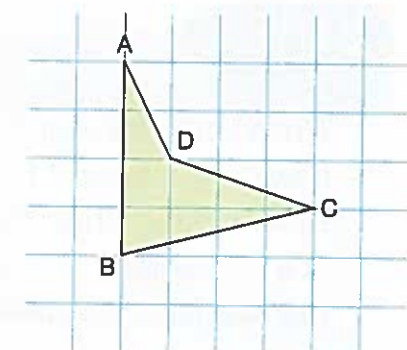
6. Copy this shape.
Use tracing paper to rotate the shape:
a) a $\frac{1}{2}$ turn clockwise about vertex E
b) a $\frac{1}{2}$ turn counterclockwise about vertex E

What do you notice about the rotation images?

7. Copy this shape on grid paper.
a) Rotate the shape about a vertex.
Describe the direction of the turn, the fraction of the turn, and the point of rotation.
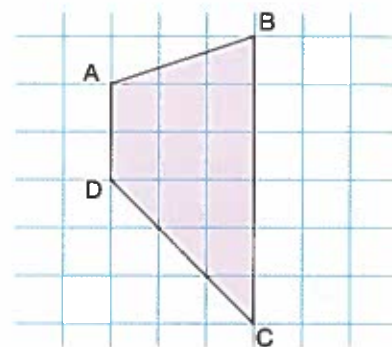Draw the image.
b) Repeat part a for a different direction.
c) Repeat part a for a different fraction.
d) Repeat part a for a different point of rotation.
Show your work.

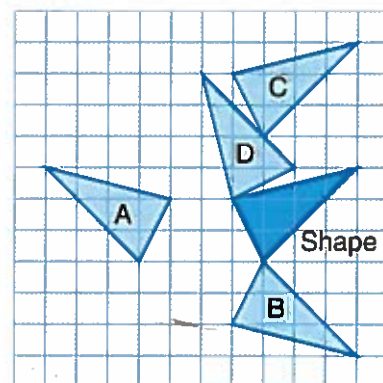**8.** Describe the transformation that moves the shape to each image.
Can you describe any movements in more than one way? Explain.
a) Image A
b) Image B
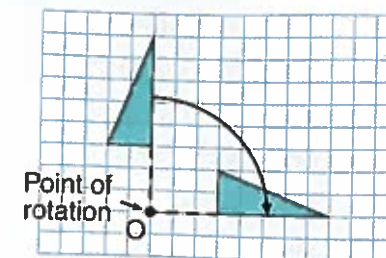**c)** Image C
d) Image D

At Home

Look for an example of each transformation.
Describe each transformation and explain how you identified it.

Exploring Different Points of Rotation

5

A shape can rotate about a point of rotation that is not on the shape.


Explore

You will need grid paper, Pattern Blocks, and a ruler.

➤ A blue Pattern Block was placed on grid paper and traced.
The block was rotated about point O and traced again.
Describe different ways the block could have moved.
Tell about the fraction of the turn, the direction, and the point of rotation.
How do the shape and its image compare?
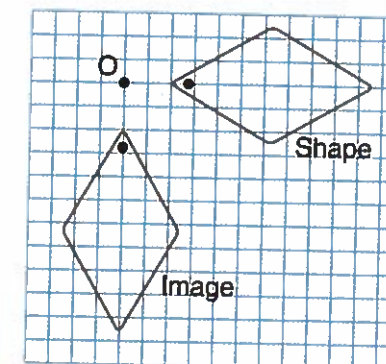
➤ Trace the blue Pattern Block on grid paper as shown.
Extend one side and mark this endpoint O.
Use point O as the point of rotation.
Choose clockwise or counterclockwise.
Rotate the block a $\frac{1}{2}$ turn about point O.
Trace its new position.
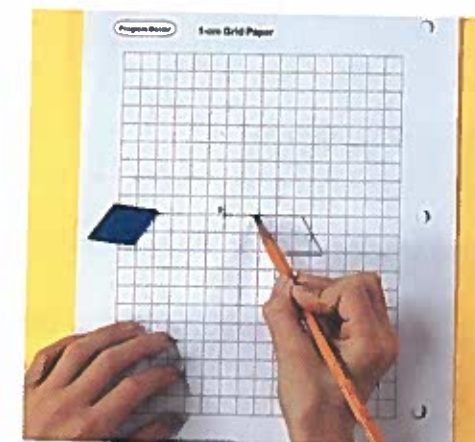
Show and Share

Exchange your tracings with a pair of students who rotated their block in the direction opposite to yours.
What do you notice? Explain.

➤ We can use tracing paper to find the image when we rotate a shape.

- Place the tracing paper so the bottom right corner is on point P.
- Trace the shape.
- Hold the tracing paper in place with your pencil at point P. Rotate the tracing paper a $\frac{1}{4}$ turn clockwise.
- Note the position of the image of the shape.
- Lift the tracing paper and draw the image in place. Label the image.
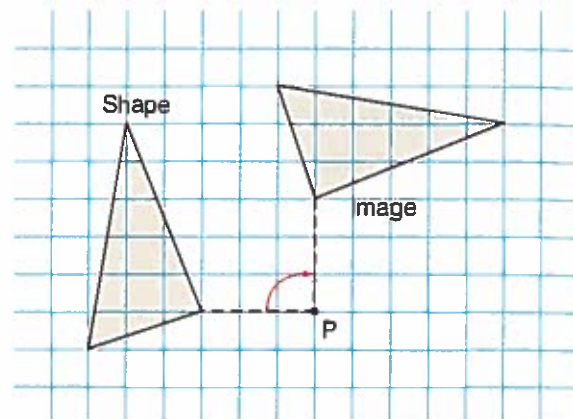
➤ We can predict the position of the image formed when we rotate a shape. Visualize the shape as a flag whose pole joins any vertex to the point of rotation. The pole rotates, but its length does not change.

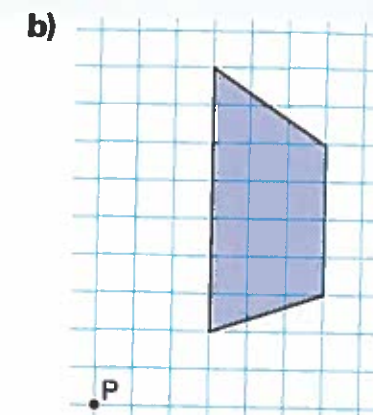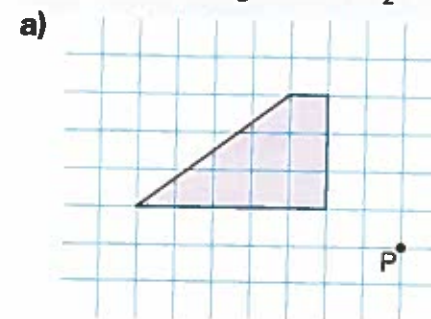a $\frac{3}{4}$ turn counterclockwise about P
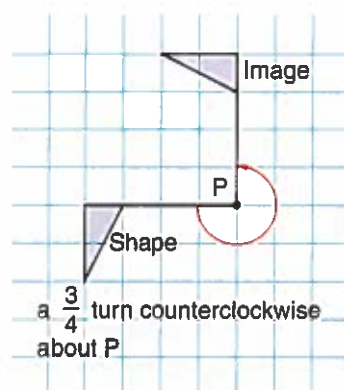
Practice

Use tracing paper when needed.

1. Copy each rectangle and point P on grid paper. Draw each image after a $\frac{1}{4}$ turn clockwise about point P.
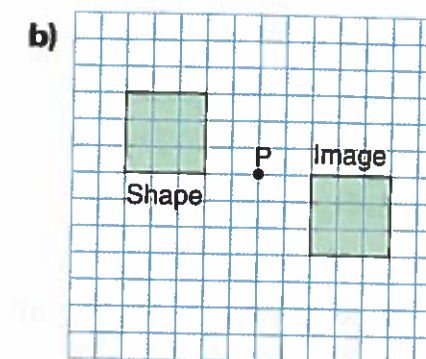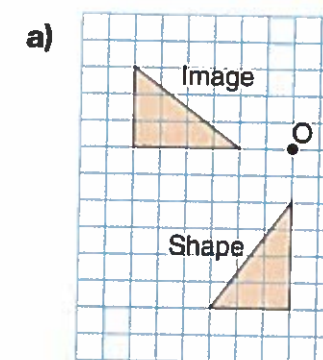
    a)

    b)
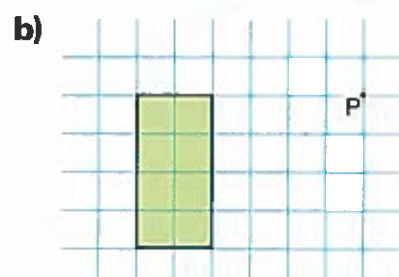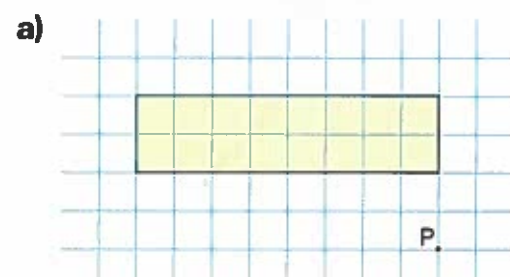
2. Copy each trapezoid and point P on grid paper. Draw each image after a $\frac{1}{2}$ turn clockwise about P.

    a)
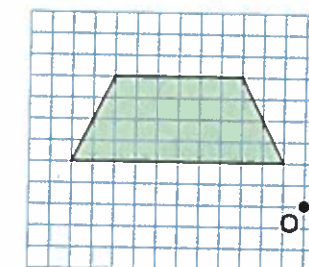
    b)

3. Describe each rotation. Include:
    - the fraction of the turn
    - the point of rotation
    - the direction

    a)

    b)

4. Copy this trapezoid and point O on grid paper.
    a) Draw the image after a $\frac{1}{4}$ turn clockwise about point O.
    b) Draw the image after a $\frac{1}{2}$ turn counterclockwise about point O.
    c) How can you tell if you have drawn the correct images?

5. Draw a quadrilateral on grid paper. Choose a point outside the quadrilateral. Rotate the quadrilateral about the point you chose. Draw its rotation image. Describe the rotation.

Reflect

When you see a shape and its rotation image, how can you tell if the point of rotation is on or off the shape?

Transformations on a Computer

Use dynamic geometry software.
Open a new sketch.
Check that the distance units are centimetres.
Display a grid.

> If you need help at any time, use the Help menu.

Translating a Shape

Construct a rectangle. Select the rectangle.
Translate the rectangle 4 squares left and 2 squares down.
Print the rectangle and its translation image.

Reflecting a Shape

Construct a triangle.
Select one side of the triangle as the line of reflection.
Select the triangle. Reflect it in the line of reflection.
Print the triangle and its reflection image.
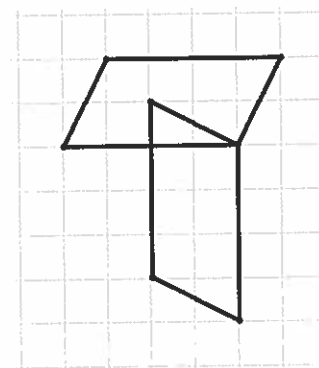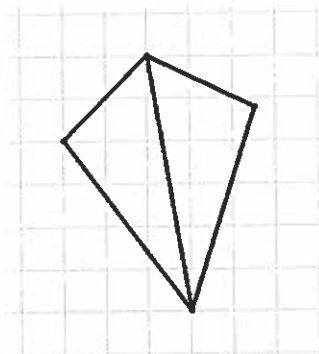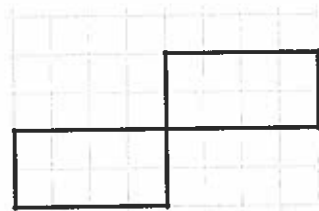
Rotating a Shape

Construct a parallelogram.
Select a vertex of the parallelogram as the point of rotation.
Select the parallelogram. Rotate it a $\frac{1}{4}$ turn counterclockwise.
Print the parallelogram and its rotation image.

Identifying Rotations

Work with a partner. Take turns.

➤ Construct a shape on the grid.
  Choose a rotation and construct the rotation image.
  Print the picture.
  Have your partner identify the rotation.
  Remind her to include:
  • the point of rotation
  • the fraction of the turn
  • the direction of the turn

➤ Repeat the steps above for different points of rotation,
  different fractions of a turn, and different directions.

Predicting the Image

Work with a partner.
Take turns.

➤ Construct a shape on the grid.
  Print the shape.
  Choose a translation.
  Tell your partner what it is.
  Have your partner predict where
  the translation image will be.
  Translate the shape
  and draw its image.
  Print the shape and its image to verify
  your partner's prediction.

➤ Repeat the steps above for a reflection.

➤ Repeat the steps above for a rotation.

Identifying a Transformation

Work with a partner.
Take turns.

➤ Choose a transformation.
  Construct a shape and its image.
  Have your partner look at the screen and
  identify the transformation.

➤ Repeat the steps above for different
 transformations and different shapes.

Reflect

How does each shape and its image compare?
Do the comparisons match those you made from pictures
you drew in earlier lessons? Explain your ideas.

1. Copy the shape on grid paper.
   a) Translate the shape in any direction you like.
      Draw its translation image.
   b) Draw a line of reflection.
      Draw the reflection image.
   c) Choose a point of rotation and a fraction of a turn.
      Rotate the shape and draw its rotation image.
   d) Describe the position and orientation of each image in parts a, b, and c.
   How does each description help you identify the transformation?
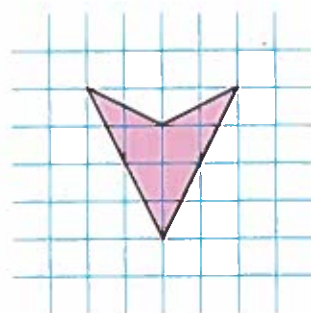
2. Draw a shape on grid paper.
   a) Translate the shape any way you like.
      Draw its translation image.
      Record the translation.
      Include each direction and the number of squares moved.
   b) Reflect the shape.
      Draw its reflection image.
      Label the line of reflection.
      Find how far the shape and its image are from this line.
   c) Rotate the shape.
      Draw its rotation image.
      Describe the rotation.
      Include the direction of the turn, the fraction of the turn, and the point of rotation.
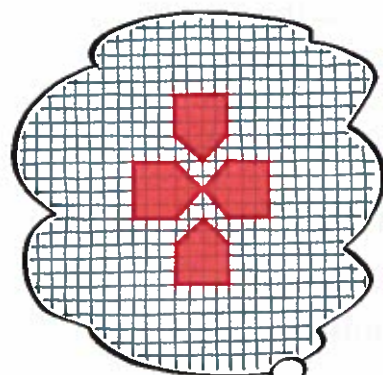
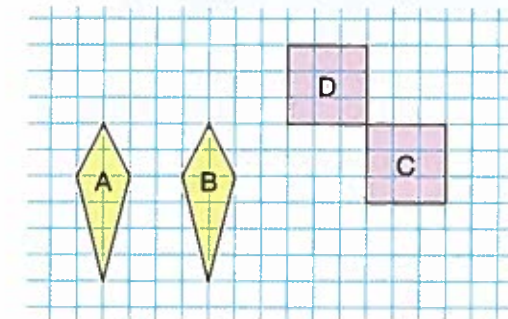3. Describe a transformation that would move shape A to each image.
   a) Image B   b) Image C
   c) Image D   d) Image E
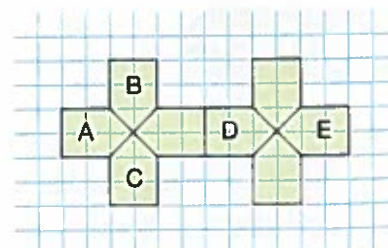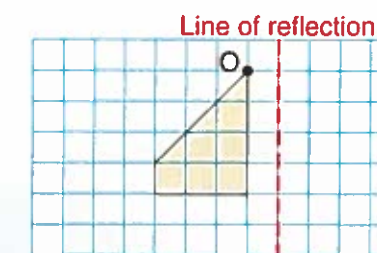
4. Describe the translation that moves:
   a) Shape B to Image A
   b) Shape D to Image C

5. In question 4, which other transformation would move each shape to its image?

6. Copy this shape on grid paper.
   Predict the position of the image after each transformation below.
   Draw each image to check your prediction.
   a) a reflection in the line of reflection
   b) a translation 3 squares right and 4 squares up
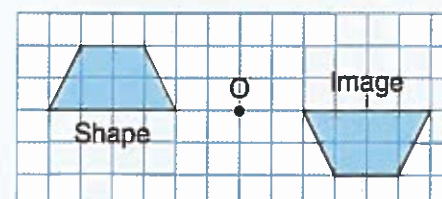   c) a $\frac{1}{4}$ turn counterclockwise about O

Line of reflection

7. Copy this triangle and point O on grid paper.
   Draw the image after a $\frac{1}{4}$ turn clockwise about O.
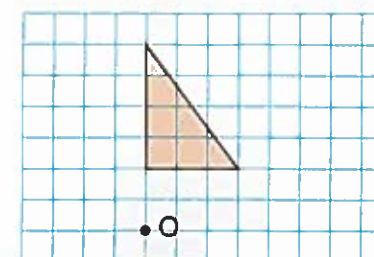
8. Describe the transformation that moves the shape to its image.

Shape   O   Image
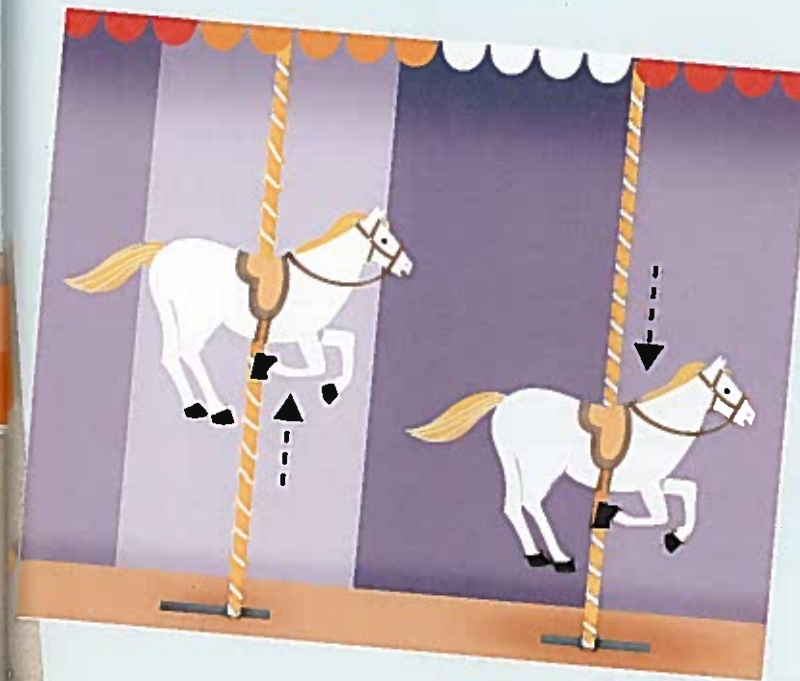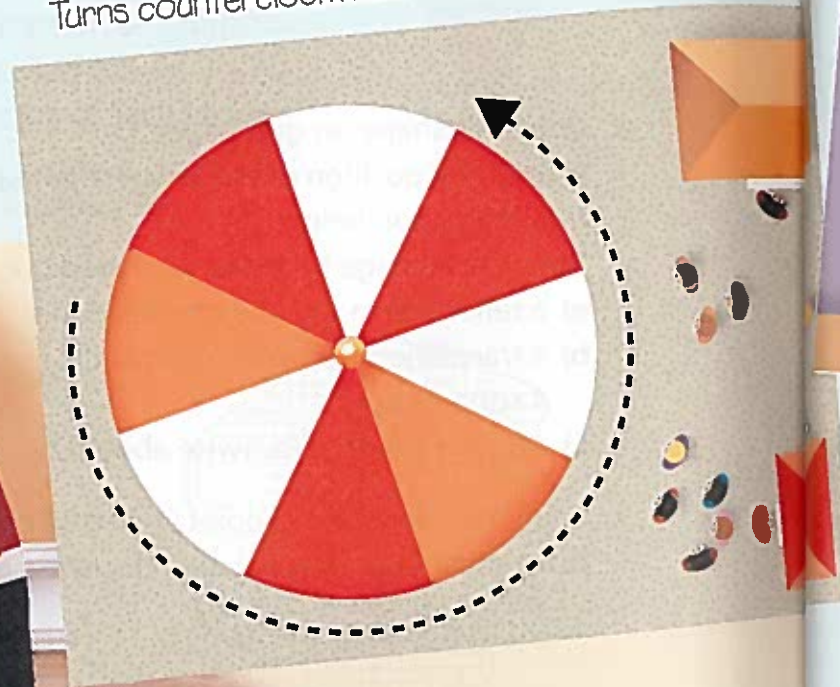
Learning Goals

UNIT 8

☑ translate, reflect, and rotate a shape
☑ draw and describe images after transformations
☑ identify a transformation

Unit Problem

At the Amusement Park

Design a ride for an amusement park. The ride must move people in at least 2 *different* ways.

Turns counterclockwise.

Moves up and down.

Think about how you will present your ride to the class.

Will you
- make a drawing?
- make a model?
- write about it?
- talk about it?

How does your ride move people in at least 2 different ways?

Reflect on Your Learning

What did you learn about how shapes can move? Use words and pictures to explain.